Abstract. Strange quark matter in a color flavor locked (CFL) state can be the true ground state of hadronic matter for a much wider range of the parameters of the model (the gap of the QCD Cooper pairs Δ, the strange quark mass m_s and the Bag Constant B) than the state without any pairing. We review the equation of state (EOS) of CFL strange matter and study the structure of stellar objects made up of this phase, highlighting the novel features of the latter. Although the effects of pairing on the equation of state are thought to be small, we find that CFL stars may be in fact much more compact than strange stars (SS). This feature may be relevant in view of some recent observations claiming the existence of exotic and/or deconfined phases in some nearby neutron stars (NS).

Key words: Stars: neutron – Equation of state-

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High-density QCD pairing in compact star structure

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1. Introduction

The study of a hypothetical stability of strange quark matter (SQM), put forward in Witten's (1984) seminal paper and a few important precursors (Bodmer 1971; Terazawa 1979; Chin and Kerman 1979) has entered its third decade. Nothing less than the nature of the true ground state of hadronic matter is being questioned and in fact, within simple models, the hypothesis of a stable form of cold catalyzed plasma was shown to be tenable. Following the pioneering works, a general calculation of strange matter by Farhi and Jaffe (1984) in the framework of the MIT Bag model of confinement (Degrand et al. 1975; Cleymans, Suhonen and Gavai 1986) identified the so-called "windows of stability", or regions in the plane $m_s - B$ inside which the stability of SQM can be realized. Other models of confinement have been worked out to find a quite ample range of conditions for SQM to be absolutely bound (Zhang and Ru-Ken Su 2002 and references therein; Hanauske et al. 2001). However, there is a consensus that the issue of the availability of a $\sim 1\,\%$ binding energy difference for SQM to be bound is ultimately an experimental matter.

While the search for SQM in laboratory and astrophysical environments beyond their present limits continues, important theoretical developments have taken place. The most sound is probably the revival of interest in pairing interactions in dense matter, a subject already addressed in the early 1980s (Bailin and Love 1984) and revived a few years ago with new calculations of the pairing energy and related physics. It is now generally agreed (Alford, Rajagopal and Wilczek 1999b; Rapp et al. 2000; Rajagopal and Wilczeck 2000) that (at least for very high densities) the color-flavor locked (CFL) state, with equal numbers of u, d and s quarks is likely to be the ground state of strong interactions, even if the quark masses are unequal (Alford, Berges and Rajagopal 1999a; Schäfer and Wilczek 1999). The equal number of flavors is enforced by the symmetry of the state, and thus electrons are absent because the mixture is automatically neutral (Rajagopal and Wilczeck 2001; Steiner, Reddy and Prakash 2002).

Long awaited by theoretical physicists, the high-performance of the space X-ray missions Chandra and XMM (Weisskopf 2002; Becker and Aschenbach 2002) enabled unprecedented studies of imaging and spectra of selected neutron stars with the aim of determining the masses and radii, perhaps the most simple forms of (indirectly) investigating the nature of high density matter. Adopting General Relativity as a framework, a comparison of the static models generated by integration of the Tolman-Oppenheimer-Volkof equations with observed data is expected to give information about the equation of state, and possibly other effects such as rotation, magnetic atmospheres and so on. A wonderful example of the pre-space determinations is the mass of the binary pulsar PSR 1913+16, accurate to several decimal places (van Kerkwijk 2001). Other methods based on combinations of spectroscopic and photometric techniques have been recently proposed. At least one X-ray source is significantly above the centroid of the binary distribution $\sim 1.4\,M_\odot$; namely Vela X-1 for which a value of $1.87^{+0.23}_{-0.17}\,M_\odot$ has been obtained (van Kerkwijk 2001). Recently, not only the masses have been determined, but also indications of the radii became available, suggesting a very compact structure. For instance, claims of high compactness have been made from the analysis of the binary Her X-1 (Li, Lai and Wang 1995; Dey et al. 1998) with $M = 0.98 \pm 0.12 M_\odot$ and $R = 6.7 \pm 1.2$ km, and of the isolated nearby RX J185635-3754 (Pons et al. 2002) with $M \approx 0.9 M_\odot$ and $R \approx 6$ km. In both cases, the results have been revisited and challenged by other groups (Reynolds et al. 1997; Kaplan, van Kerkwijk and Anderson 2001) who in turn found figures around the expected for conventional neutron star models. This stresses the cautionary remarks made by several researchers about the high-compactness objects and guarantees further studies, already undertaken in most cases. Li et al. (1999b) have also added the source 4U 1728-34 to the candidate list, showing that conventional accretion models indicate a very compact source in the mass-radius plane. As always, the actual distance to the source is a matter of concern. Needless to say, these results have yet to be confirmed carefully. Nonetheless, it is worthwhile to consider the possibility that at least some compact stars

Fig. 1. The dashed lines show the equation of state for CFL strange matter for B=75 MeV fm^{-3}, $m_s = 150$ MeV, and two different values of the gap Δ as indicated in the figure. The solid line corresponds to SQM without pairing. Note the change of stiffness according to the value of Δ, as discussed in the text.

$$-\frac{3}{\pi^2}\Delta^2\mu^2 + B, \tag{1}$$

where $3\mu = \mu_u + \mu_d + \mu_s$, μ_i being the chemical potential of the i-species. The common Fermi momentum $\nu = (\mu_i^2 - m_i^2)^{1/2}$ is given by

$$\nu = 2\mu - \left(\mu^2 + \frac{m_s^2}{3}\right)^{1/2}. \tag{2}$$

The thermodynamic quantities, such as the pressure P, the baryon number density n_B, the particle number densities n_i, and the energy density ε can be easily derived at $T = 0$ and read (Lugones and Horvath 2002)

$$P = -\Omega_{CFL}, \tag{3}$$

$$n_B = n_u = n_d = n_s = \frac{(\nu^3 + 2\Delta^2\mu)}{\pi^2}, \tag{4}$$

$$\varepsilon = \sum_i \mu_i n_i + \Omega_{CFL} = 3\mu n_B - P. \tag{5}$$

In this approach we shall treat the values of B, m_s and Δ (possibly as high as $\sim 100\,MeV$) as free constant parameters. The full dependence of Ω_{CFL} on m_s has been known for years and certainly complicates the evaluation of the equation of state, which must be treated numerically. However, we have recently worked out an approximation to the order m_s^2 which has the main advantage of keeping the equation of state very simple, yet useful for most calculations, while at the same time highlights the effect of each parameter of the model (see Lugones and Horvath 2002 for details).

The stiffness of the EOS is relevant for stellar models since it has a direct impact on the compactness of the stellar configurations. As discussed by Lugones and Horvath (2002), whenever Δ is higher than $m_s/2$, the EOS is stiffer than the unpaired SQM (that is, produces more pressure for a given energy density). Since the actual value of Δ is not well known, we may expect either case, a stiffer or a softer EOS (for a given B).

Fig. 3. The mass-radius relation for CFL strange stars for B =115 MeV fm^{-3}, $m_s = 150$ MeV, $\Delta = 100$ MeV on the left, and for B = 70 MeV fm^{-3}, $m_s = 150$ MeV, $\Delta = 100$ MeV on the right. The crosses indicate the models of maximum radius for each set of parameters, while the asterisks indicate the models with the maximum mass (see Figs 4-7 for more details).

lower than 939 MeV at zero pressure. Eq. (7) sets the right side boundary of the window while the left side boundary is imposed by the minimum value B = 57 MeV. The window is greatly enlarged for increasing values of Δ with respect to the original SQM calculations which do not include any pairing (see for example, Fig. 1 of Farhi and Jaffe 1984). We have emphasized elsewhere that the pairing gap Δ may be important for a "CFL strange matter" state (Lugones and Horvath 2002; see also Madsen 2001 for a similar view).

3. Structure of color flavor locked stars

The study of the mass-radius relation for compact stars is a widely known tool for testing the existence of different phases of matter inside NSs. It is also well-known that simultaneous observational data on masses and radii can impose important constraints on the high density equations of state. Up to now, almost all of the measured masses of "neutron stars" clustered within a narrow range around 1.4 $M_\odot$. It has been conjectured that this mass scale may be due to the fact that neutron stars are formed in the gravitational collapse of supernovae and come just from the iron cores of the pre-supernovae. However, nothing fundamental precludes smaller neutron stars from existing provided some mechanism capable of creating them operates. Because small masses are not expected in core collapse models, they hold the potential for discriminating among possible equations of state. This is particularly significant given the recent claims of very low-mass/low-radii objects. As already mentioned, at least two sources, Her X-1 and RX J1856-37, are candidates for high compactness. They have been claimed to have radii $\sim 7km$ and masses around $1M_\odot$ (Li et al. 1995; Pons et al. 2002).

Fig. 5. The same as the previous figure but for fixed Δ and different values of m_s.

range of the parameters, which should be limited by observational arguments much in the same way as done for strange matter models.

4. Discussion

We have addressed in this work stellar models constructed with the simplest version of a "CFL strange matter" phase. The CFL phase at zero temperature has been modelled as an electrically neutral and colorless gas of quark Cooper pairs, in which quarks are paired in such a way that all the flavors have the same Fermi momentum and

and B = 153 MeV fm^{-3} respectively) the values obtained for M_{max} and R_{max} from Figs. 4-7 are quantitatively in agreement (even if the latter value has to be extrapolated if needed in these Figs.). Moreover, a variety of CFL strange star models can be explored by varying the parameters, although, for example, a large variation in m_s does not result in substantial modifications of M_{max} and R_{max}.

Fig. 7. The same as the previous figure but for fixed Δ and different values of m_s.

hence the same number density (Rajagopal and Wilczeck 2001). Since the strange quark is massive, some energy must be spent in order to keep a common Fermi momentum for all three flavors. However, more than that is gained from the energy gap of the pairing. The model allows CFL strange matter to be the true ground state of strong interactions for a wide range of the parameters B, m_s and Δ. It is remarkable that the strange matter hypothesis may be quite *boosted* by pairing interactions. We have explored the mass-radius relation for all the values of the parameter space of the EOS that give absolutely stable CFL strange matter. Very compact configurations are found that could help to explain the recently claimed compactness of a few neutron stars. Also, for some parameter choices the models are massive and extended.

A feature that has been largely explored and debated in the context of strange star models is the existence of a normal matter crust (Zdunik 2002). Bare quark surfaces may alter drastically the ability of radiating photons, and thus they may produce interesting signatures for their identification (Page and Usov 2002). Conversely, a normal matter crust (held in mechanical equilibrium by the electrostatic potential at the surface) may hide most of the features of exotic matter. By its very construction, it is clear that (in striking contrast with the well-studied SS) no crust could be present in the case of CFL strange stars. This is directly related to the absence of electrons in the mixture (Rajagopal and Wilczek 2001), and thus to the absence of an electrostatic potential to prevent normal matter from being converted to the stable CFL state. CFL strange stars must have bare surfaces within these models. This in turn means that the transport properties of the state, which are still being explored (see, for example Shovkovy and Ellis 2002; Jaikumar, Prakash and Schafer 2002; Reddy, Sadzikowski and Tachibana 2003), hold the clue to the identification of these compact stars. We expect the photon emission properties of CFL strange stars to resemble those of bare SS. Briefly, pairing effects should affect the plasma frequency ω_p through the baryon number density as a small correction of order $\mu\Delta^2$ so that ω_p will not be very different from the $\sim 20MeV$ expected for bare SS . The equilibrium photon radiation will show a very hard spectrum and a tiny luminosity, making CFL strange stars very difficult to detect. On the other hand, while the thermal emission of photons from the bare quark surface of a hot strange star (mainly by electron-positron pair production) has been shown to be much higher than the Eddington limit (Page and Usov 2002), this would not be the case for CFL SS since no electrons will be present at the surface.

As already mentioned, the fact of considering CFL SQM to be absolutely stable together with the absence of electrons in the mixture precludes the existence of a crust of normal matter, then CFL strange stars are bare by construction. In a more general approach, Alford and Reddy (2002b) found mass-radius relations with essentially the same shape as that for neutron stars made up of normal nuclear matter when considering a wider range of parameters in which "hybrid" stars are allowed. This corresponds to a large part of the parameter space and not only do they find a normal matter envelope for all these models, but also the composition of the less massive objects becomes pure nuclear matter, as expected. In addition, as pointed out in Section 3, a subset of the models of Alford and Reddy (2002) correspond to our "CFL strange stars", and the full range of these self-bound models can be obtained with the aid of Figs. 4-7. The self-bound CFL state may be compared to stable diquark states, which have been found to produce very similar stellar structural properties than CFL strange matter within completely different models (Horvath 1993; Horvath and de Freitas Pacheco 1998; Lugones and Horvath 2003). We finally note that, due to the presence of m_s and Δ in the EOS, a linear form $P(\rho)$ is obtained in particular cases only, therefore the scaling behavior of the Tolman-Oppenheimer-Volkov equations that allows the construction of simple expressions for the maximum masses and radii is lost for these CFL stars. The relevance of this modelling to understand actual observed stars is not clear as yet, but certainly the latter will continue to advance to the point at which this and other questions will be answered with confidence.

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References

Alford M., Berges J. and Rajagopal K. 1999a, Nucl. Phys. B 558, 219

Alford M., Rajagopal K. and Wilczek F. 1999b, Nuc. Phys. B 537, 433

Alford M., Rajagopal K., Reddy S. and Wilczeck F. 2001, Phys. Rev. D 64, 074017

Alford M. and Rajagopal K. 2002a, hep-ph/0204001

Alford M. and Reddy S. 2002b, nucl- th/ 0211046

Bailin D. and Love A. 1984, Phys. Repts. 107, 325

Becker W. and Aschenbach B. 2002, in Proceedings of the 270.WE-Heraeus Seminar on Neutron Stars, Pulsars and Supernova Remnants, Jan 21-25, 2002, Physikzentrum Bad Honnef, eds W. Becker, H. Lesch and J. Truemper

Bodmer A. 1971, Phys. Rev. D 4, 1601

Cheng K. S., Dai Z. G., Wai D. M. and Lu T. 1998, Science 280, 407

Chin S. and Kerman A. K. 1979, Phys. Rev. Lett. 43, 1292

Cleymans J., Gavai R. V. and Suhonen E. 1986, Phys. Rep. 4, 217

Degrand T., Jaffe R. L., Johnson K. and Kiskis J. 1975, Phys. Rev. D 12, 2060

Dey M., Bombaci I., Dey J., Ray S. and Samanta B.C. 1998, Phys. Lett. B 438, 123; Addendum 1999, Phys. Lett. B 447, 352; Erratum 1999, Phys. Lett. B 467, 303.

Farhi E. and Jaffe R. L. 1984, Phys. Rev. D 30, 2379

Hanauske M., Satarov L.M., Mishustin I.N., Stoecker H. and Greiner W. 2001, Phys.Rev. D 64, 043005

Hong D. K. , Hsu S. D. H. and Sannino F. 2001, Phys.Lett B 516, 362

Horvath J. E. 1993, Phys. Lett. B 242, 419

Horvath J. E. and de Freitas Pacheco J. A. 1998, Int. J. of Mod. Phys. D 7, 19

Jaikumar P., Prakash M., Schafer T. 2002, Phys. Rev. D 66, 063003

Kaplan D. L., van Kerkwijk M. H., Anderson J. 2001, astro-ph/0111174

Li X. -D., Lai Z. -G. and Wang Z. -R. 1995, Astron. Astrophys. 303, L1

Li X., Bombaci I., Dey M., Dey J. and van den Heuvel E. P. J. 1999a, Phys. Rev. Lett. 83, 3776.

Li X., Ray S., Dey J., Dey M. and Bombaci I. 1999b, Astrophys. J. 527, L51

Lugones G. and Horvath J. E. 2002, Phys. Rev. D 66, 074017

Lugones G. and Horvath J. E. 2003, Int. Jour. Mod. Phys. D 12, to be published, astro-ph/0203069

Madsen J. 2001, Phys. Rev. Lett. 87, 172003

Nakamura T. 2002, astro-ph/0205526

Page D. and Usov V. V. 2002, Phys. Rev. Lett. 89, 131101

Pons J., Walter F. M., Lattimer J. M., Prakash M., Neuhäuser R. and Penghui An. 2002, Astrophys. J., 564, 981.

Rajagopal K. and Wilczeck F. 2000, hep-ph/0011333

Rajagopal K. and Wilczeck F. 2001, Phys. Rev. Lett. 86, 3492

Rapp R., Schäfer T., Shuryak E. V. and Velkovsky M. 2000, Annals Phys. 280, 35

Reddy S., Sadzikowski M. and Tachibana M. 2003, Nucl.Phys.A 714, 337

Reynolds A.P., Quaintrell H., Still M.D., Roche P., Chakrabarty D. and Levine S.E. 1997, MNRAS, 288, 43.

Schäfer T. and Wilczek F. 1999, Phys. Rev. D 60, 074014

Shovkovy I. A. and Ellis P. J. 2002, hep-ph/0211049

Steiner A. W., Reddy S. and Prakash M. 2002, Phys.Rev. D 66, 094007

Terazawa H. 1979, INS-Report 336

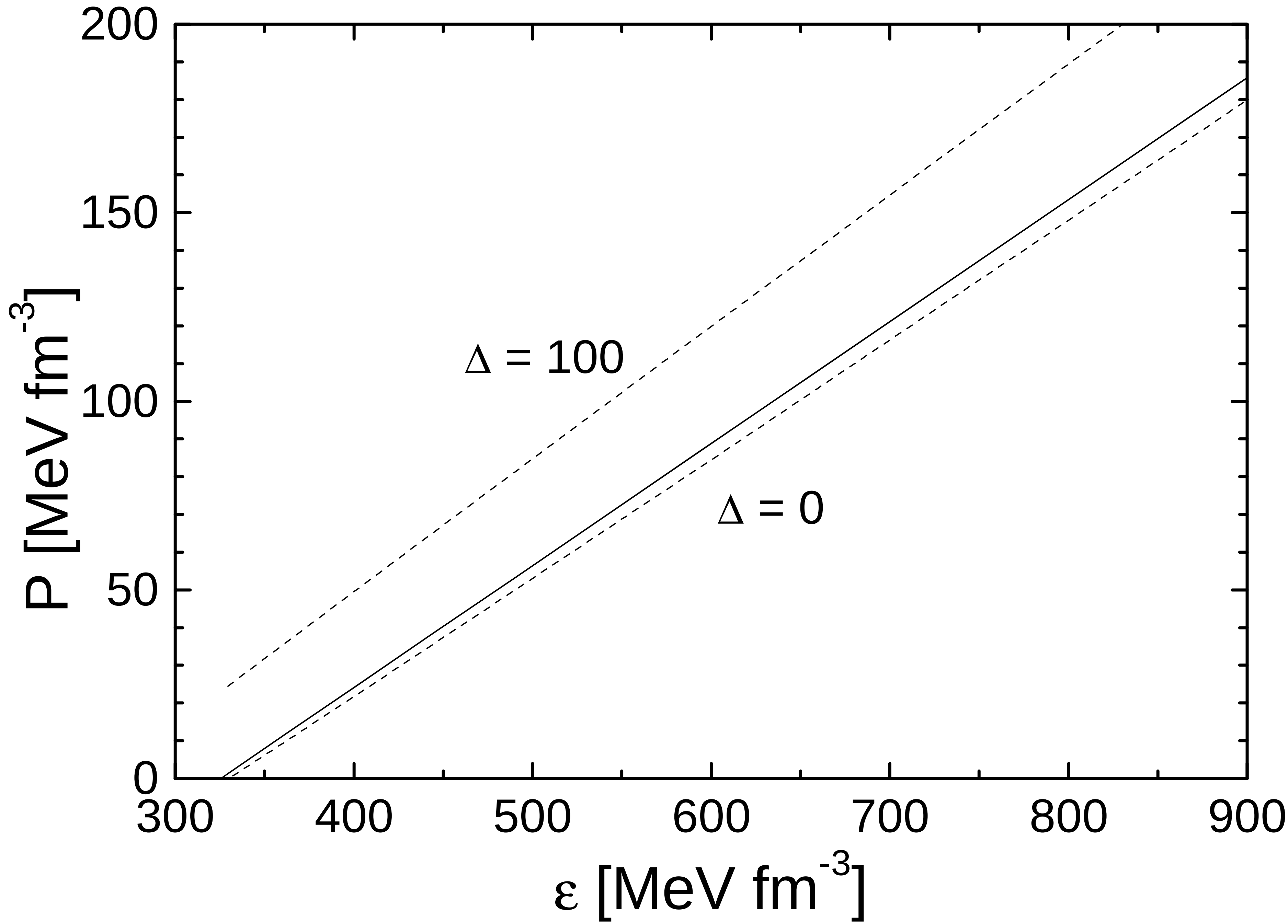
van Kerkwijk M.H. 2001, in Proceedings of the ESO Workshop on Black Holes in Binaries and Galactic Nuclei, eds. L. Kaper, E.M. van den Heuvel and P.A. Woudt (Springer-Verlag, 2001).

Weisskopf M.C. 2002, in Proceedings of the 270. WE-Heraeus Seminar on Neutron Stars, Pulsars and Supernova Remnants, Jan. 21-25, 2002, Physikzentrum Bad Honnef, eds W. Becker, H. Lesch and J. Truemper

Witten E. 1984, *Phys. Rev. D* **30**, 272

Zdunik J. L. 2002, *Astron.Astrophys.* 394, 641

Zhang Y. and Ru-Ken Su 2002, Phys. Rev. C 65, 035202



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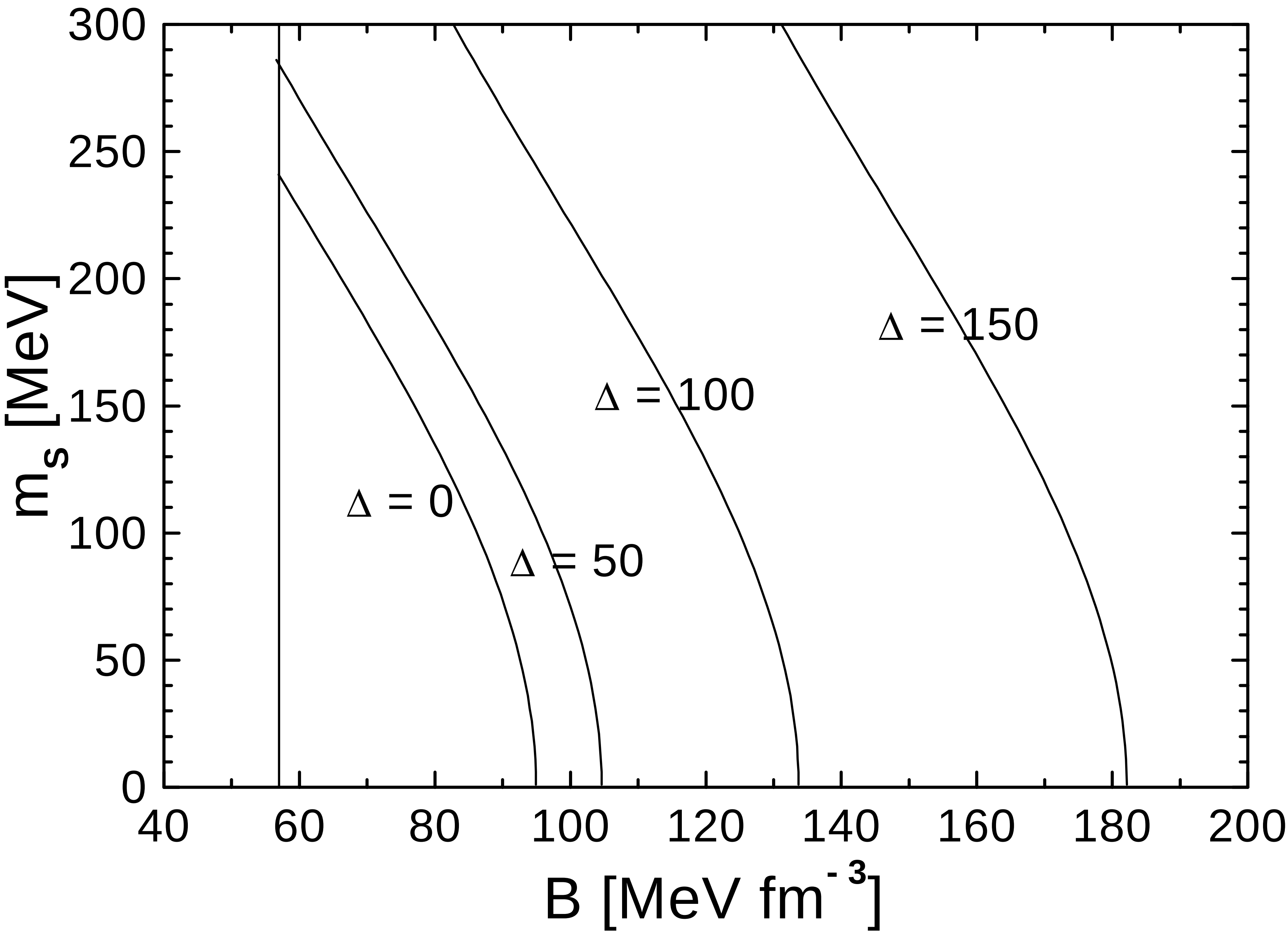

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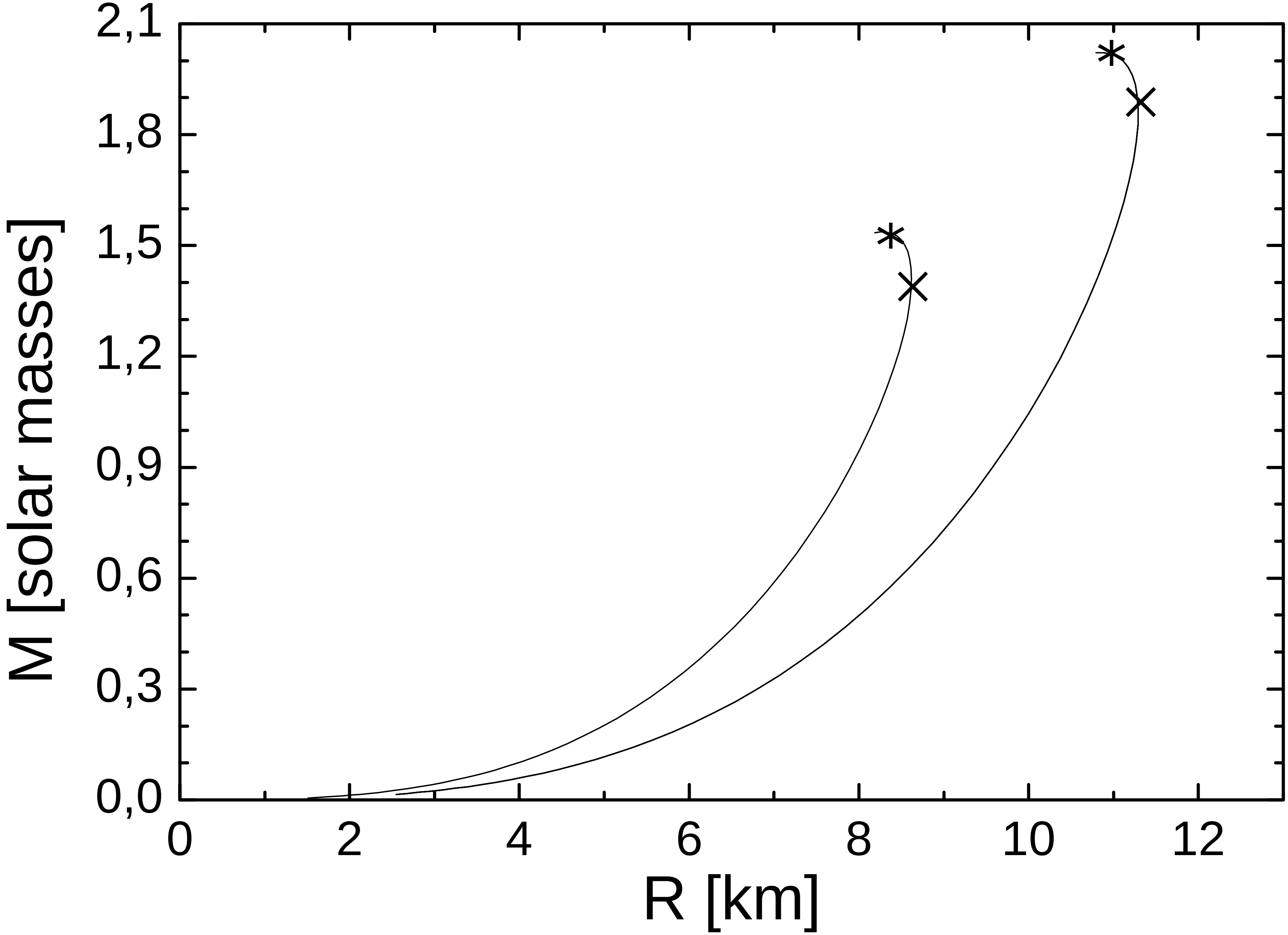
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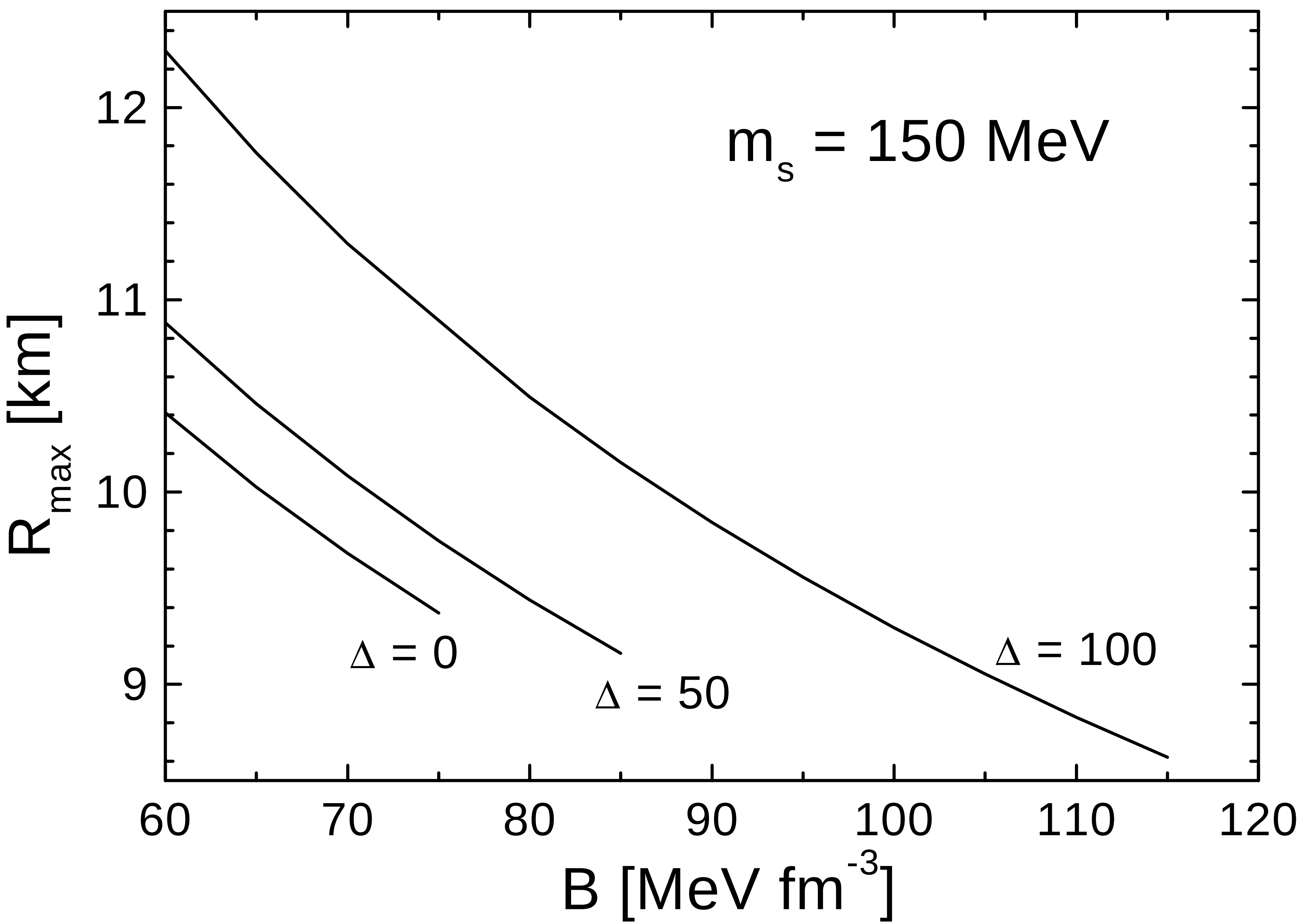

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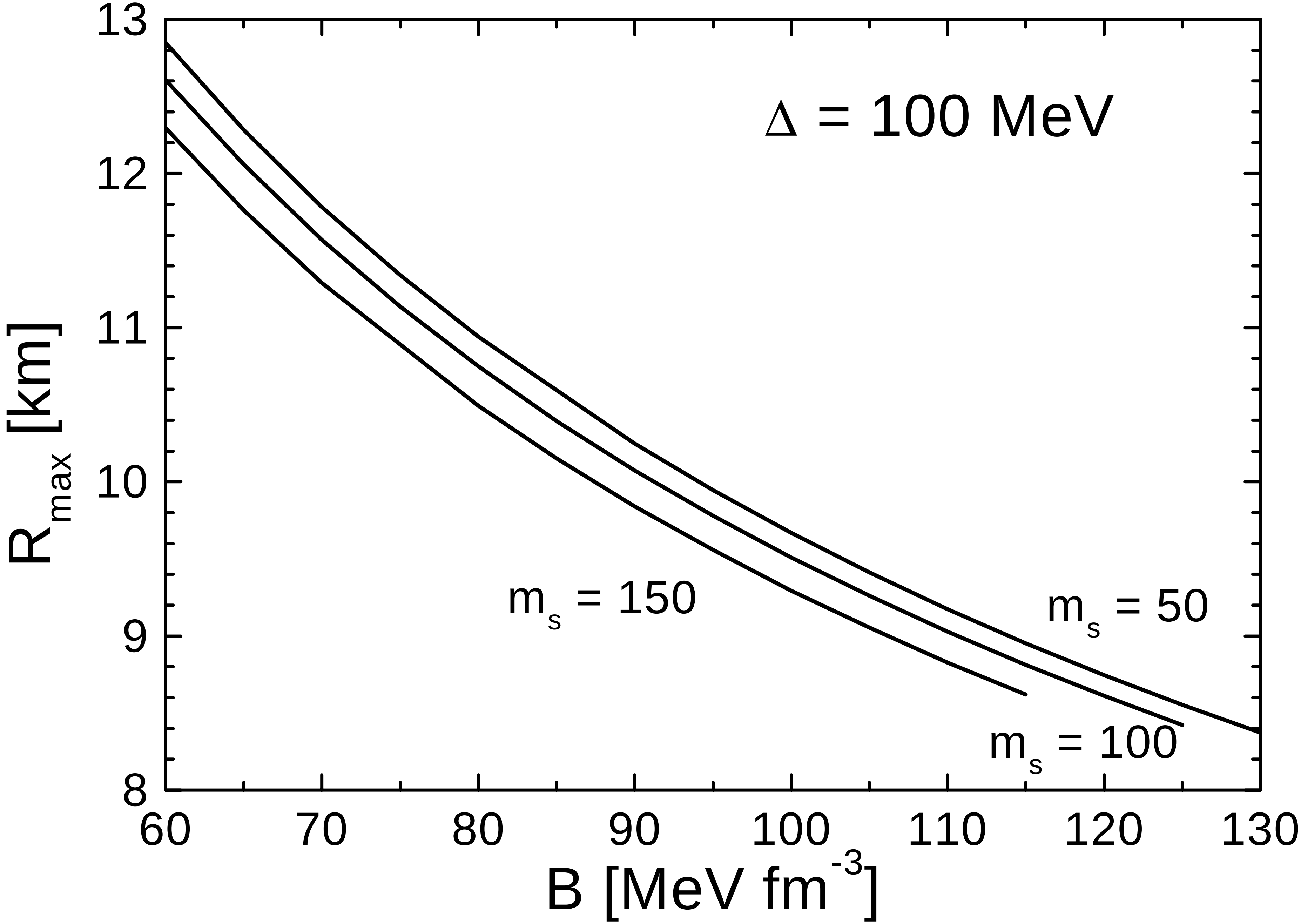

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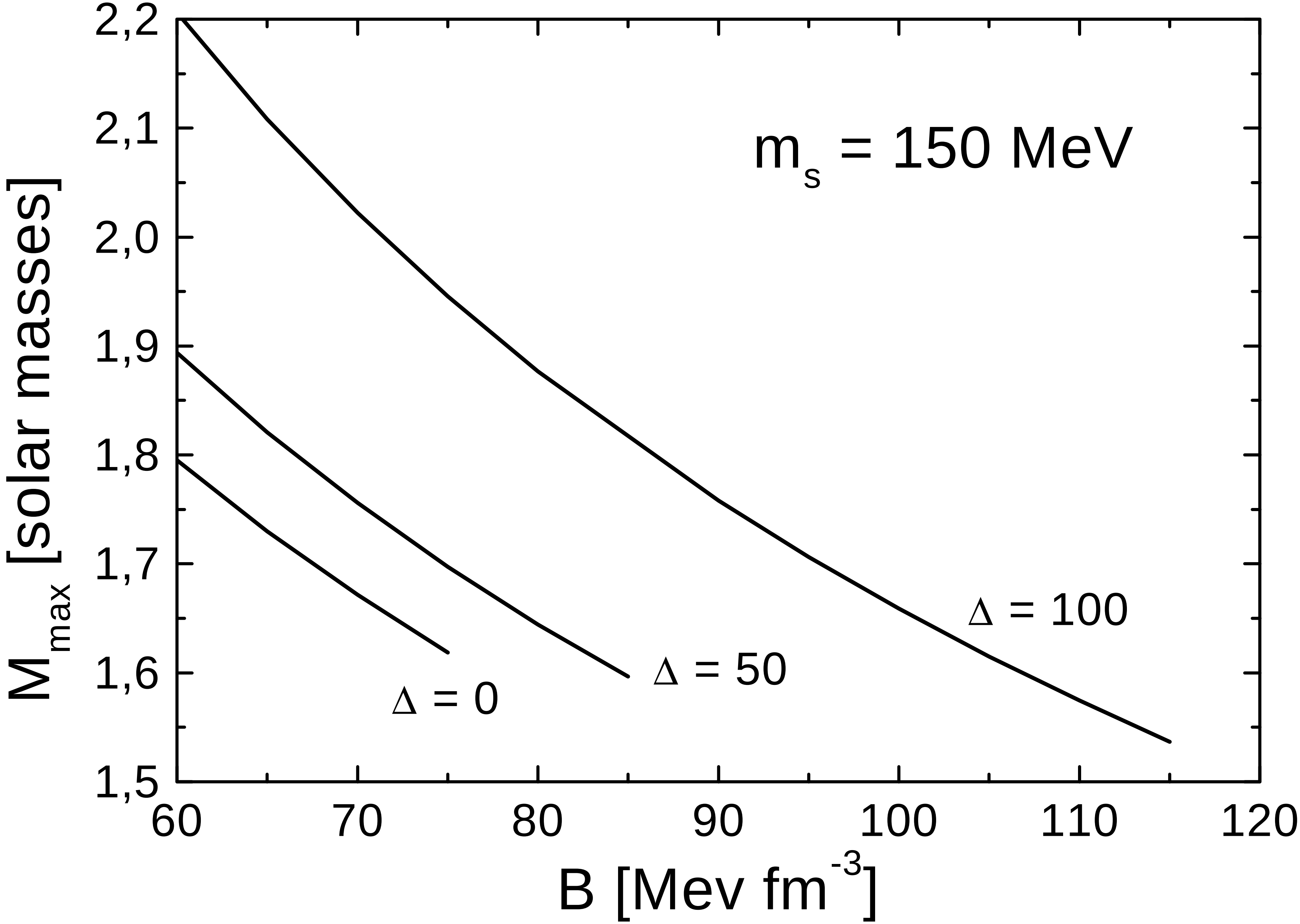

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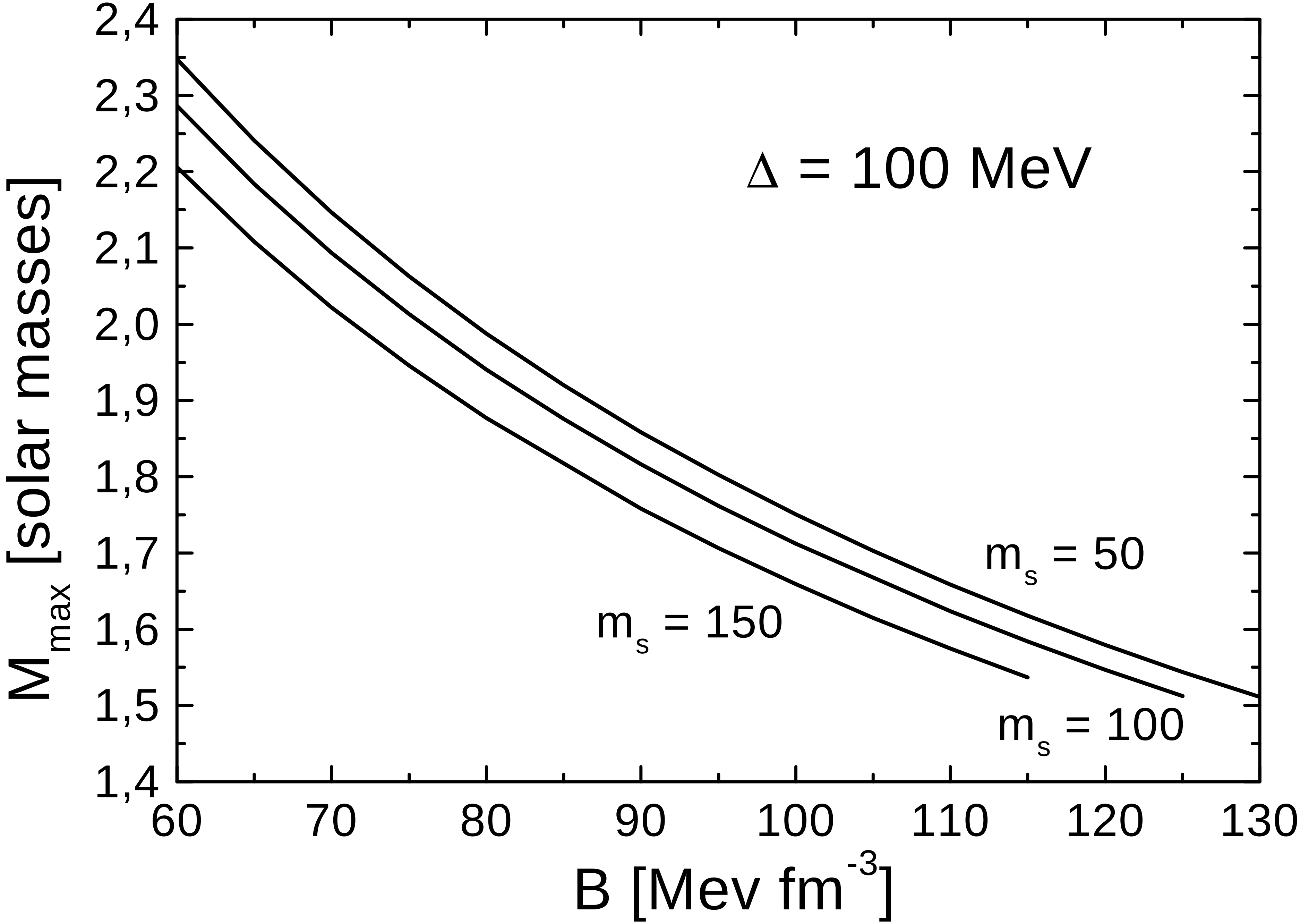
M_{max} [solar masses]
B [Mev fm^{-3}]



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